TUGR vs. Standard Fixed Effects Regression *

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1. Introduction

This report is based on my understanding of the TUGR system as presented within the methodology and FAQ sections of the TUGR website, https://tugr.org, and in the video available at https://www.tugr.org/method, a system which, according to the video, is "the most accurate, understandable, all-inclusive, unbiased [ranking] system in golf. The core principles of the system are accuracy, inclusivity, simplicity, transparency, and trust."[1] Notwithstanding this description and a cordial and informative phone conversation with two members of the TUGR team, my attempts to acquire additional information about the mathematics underlying the system beyond that described in the video and on the website have garnered no response.

According to the video, the TUGR system is based on "the following two principles of simplicity."

1. Gather head-to-head scores, player-by-player, over the last 12 months
2. Let a powerful computer solve the matrix math and rank order players with incredible precision

It then describes the accuracy of the system as follows:

> "How do we rank order golfers across tours, tournaments, and time? Easy, with the most comparable metric in golf, head-to-head matchups.
>
> Imagine you have a friend named Chris. He plays golf with Nick quite often, and beats him, on average, by two strokes. Now, Nick has a friend named Ben who he beats on average by one stroke. Given the data we have, how could we rank order these players?
>
> Pretty easy, right? If Chris is two strokes better than Nick in head-to-head matches, and Nick is one stroke better than Ben, it doesn't take a rocket scientist to figure out that Chris is three strokes better than Ben.

In this report I formulate a fixed effects regression model that I believe is consistent with the general description of TUGR as presented in the video and on the TUGR website, especially as it relates to the following question and answer in the FAQ section:

> Q: "The matrix shows Player A shot .20 strokes lower than Player B in head-to-head match ups, but Player B is ranked higher overall. How does that work?"
>
> A: "The matrix takes into account all head-to-head scores across the entire universe of players. Player A and B crossed paths in a certain number of events, but they also played events in which the other was not present. Player B might have won or placed very high in several tournaments when Player A didn't attend. Given all the players are interconnected in the matrix, the math can triangulate to exactly how much credit Player B should get for those good performances relative to Player A. In this way, Player B might be rightfully ranked higher overall."

[1] The video is also available on YouTube at https://www.youtube.com/watch?v=4XE61r4sX54. This quote begins at 0:04 in the video.

This response seems to suggest that the TUGR system seeks a best-fit difference in expected scores of all players within the system rather than a literal average of head-to-head (H2H) scoring differences, while taking into account that players who never face each other in actual competition are linked through their common opponents. Throughout the remaining analysis, I emphasize that I do not actually know the underlying details of TUGR math. Nevertheless, my version of the TUGR system produces results consistent with the non-technical description of TUGR as presented in the video, on the website, and, especially, in response to the question above from TUGR's FAQ section. And unlike TUGR, which states, "We offer full transparency into the process,"[2] all that follows in this report is written with full transparency and could be easily replicated by anyone with reasonable programming skills who had access to the underlying data.

2. Player Skills as Estimated via Standard Fixed Effects Regression

2.1. The Broadie-Rendleman (2013) Model

Table 1 shows a panel of 18-hole scores for five players, A, B, C, D, and E, over four rounds numbered 1-4. Scores for players A, B, and C are designed to produce the same scoring differences as those for Chris, Nick, and Ben as described on the TUGR website and video, where, on average, Chris's scores are 2 strokes per round better (lower) than those of Nick, Nick's are 1 stroke per round better than those of Ben, and despite the fact that Nick and Ben never face each other in actual competition, by inference Nick's scores are 3 strokes per round better than those of Ben.

Table 1: Hypothetical 18-Hole Scores over Four Rounds in Panel Form

Player	Player Index i	Round Index j: 1	2	3	4
A	1	68	71		
B	2	71	72	73	71
C	3			75	71
D	4	67	72	72	73
E	5	67			69

Table 2 organizes the same data as it might appear in an input file, with each row or record, identified by the index k, showing the player's name, the player's identifying index value, i, an index value, j, associated with the round played, and the 18-hole score of player i in round j, denoted as $s_{i,j}$.

[2] This quote is from the "Understandable" section of the TUGR website.

Table 2: Hypothetical 18-Hole Scores over Four Rounds by Player/Round

Record Index k	Round Index j	Player Name	Player Index (i)	Score $s_{i,j}$
1	1	A	1	68
2	1	B	2	71
3	1	D	4	67
4	1	E	5	67
5	2	A	1	71
6	2	B	2	72
7	2	D	4	72
8	3	B	2	73
9	3	C	3	75
10	3	D	4	72
11	4	B	2	71
12	4	C	3	71
13	4	D	4	73
14	4	E	5	69

Following Broadie and Rendleman (2013), consider the following ordinary least squares fixed effects regression model as estimated in connection with the Table-1 scoring data:

$$s_{i,j} = \mu_i + \delta_j + \epsilon_{i,j} \tag{1}$$

In equation (1), $s_{i,j}$ is the score of player i in connection with round (or round-course combination) j, μ_i is the estimate of the mean score of player i on a "neutral" course, δ_j is the estimate of round-course effect j, and $\epsilon_{i,j}$ is the error term, with $E(\epsilon_{i,j})|i = 0$, $E(\epsilon_{i,j})|j = 0$, which imply $E(\epsilon_{i,j}) = 0$.

Note that the equality in (1) holds if all μ_i are increased by a constant and all δ_j are decreased by the same constant. To remove this arbitrary degree of freedom, δ_1 is set to zero; therefore each mean neutral score, μ_i, is an estimate of the score player i would be expected to shoot, on average, on the course as played under the conditions of round-course combination $j = 1$.

Table 3 summarizes estimated player effects (or skills) for all players, A-E. Actual estimated player effects are shown in the "Actual" column. Estimates incremental or relative to that of player A are shown in the next-to-last column, with positive values indicating skill inferior to that of player A and negative values indicating superior skill. It should be noted that if player E had been left out of the underlying data, skill estimates relative to player A would have been 2.0 and 3.0 for players B and C, respectively, just as in TUGR's video example, with the same being true if both players D and E had been left out. However, the inclusion of player E's scores in unbalanced fashion causes the relative skill estimates of players A, B, and C to change in relation to the average of their respective average H2H scoring differences.

Table 3: Estimated Player Effects

Player	Actual	Relative to Player A	E
A	68.02	0.00	1.17
B	69.44	1.41	2.58
C	69.85	1.82	3.00
D	68.69	0.66	1.83
E	66.85	-1.17	0.00

The final column of Table 3 shows player skill estimates relative to those of player E, whose estimated player effect is the lowest (best) among the five players. Although computed differently, this is the same method for reporting estimated player skill differences employed in the TUGR system.

Before proceeding to estimates of player skill based on TUGR-type H2H match-ups, it is useful to express each row, or record, in Table 2 in terms of equation (1).

$$68 = \mu_1 + \delta_1 + \epsilon_{1,1} \tag{2.1}$$
$$71 = \mu_2 + \delta_1 + \epsilon_{2,1} \tag{2.2}$$
$$67 = \mu_4 + \delta_1 + \epsilon_{4,1} \tag{2.3}$$
$$67 = \mu_5 + \delta_1 + \epsilon_{5,1} \tag{2.4}$$

$$71 = \mu_1 + \delta_2 + \epsilon_{1,2} \tag{2.5}$$
$$72 = \mu_2 + \delta_2 + \epsilon_{2,2} \tag{2.6}$$
$$72 = \mu_4 + \delta_2 + \epsilon_{4,2} \tag{2.7}$$

$$73 = \mu_2 + \delta_3 + \epsilon_{2,3} \tag{2.8}$$
$$75 = \mu_3 + \delta_3 + \epsilon_{3,3} \tag{2.9}$$
$$72 = \mu_4 + \delta_3 + \epsilon_{4,3} \tag{2.10}$$

$$71 = \mu_2 + \delta_4 + \epsilon_{2,4} \tag{2.11}$$
$$71 = \mu_3 + \delta_4 + \epsilon_{3,4} \tag{2.12}$$
$$73 = \mu_4 + \delta_4 + \epsilon_{4,4} \tag{2.13}$$
$$69 = \mu_5 + \delta_4 + \epsilon_{5,4} \tag{2.14}$$

This system of 14 equations indicates that there are nine parameters, five player effects (μ_i) and four round effects (δ_j), to be estimated, which as an OLS regression, represents a classic over-identification problem. After setting $\delta_1 = 0$, a solution for the remaining eight parameters is determined such that the sum of squared differences among the $\epsilon_{i,j}$ values is minimized. As such, each μ_i solution value is an estimate of player i's expected score in round $j = 1$, and each δ_j solution value is an estimate of how much higher or lower each player's score would be in round $j > 1$ relative to μ_i, his expected score in round 1.

2.2. Head-to-Head Fixed Effect Regression Estimates

Table 8 at the end of this paper, shows all possible head-to-head (H2H) match-ups among the five players, 50 match-ups in total. The table is organized in four sections, each showing all possible H2H match-ups per round, with each round being indexed by j. The column labeled $n\left(j\right)$ shows the total number of players who participated in round j.

The first 16 rows of Table 8 show all possible match-ups among the $n\left(j\right) = 4$ players who participated in round $j = 1$, including match-ups of each player with himself and match-ups where a player is the first in a given pair and those where the same two players are paired but in reverse order. Within the 16-row $j = 1$ section, the record number, k, takes on the values $\{1, 2, 3, 4\}$, with each value of k referring to the first, second, third, and fourth scoring records as summarized in Table 2.

For a given match-up, k_1 refers to the Table-2 record number of the first player in the match-up, while k_2 refers to the record number of the second player. The first and second players' names are shown in the two columns labeled "First Player Name (k_1)" and "Second Player Name (k_2)," while their respective player index values are shown in the "First Player Index (k_1)" and "Second Player Index (k_2)" columns. Finally, the 18-hole scores of the first and second player in each match-up are shown in the $s\left(k_1\right)$ and $s\left(k_2\right)$ columns. Although k_1 and k_2 take on the values $\{1, 2, 3, 4\}$, player names and their respective index values take on the values {A, B, D, E} and $\{1, 2, 4, 5\}$, because player C, with index $i = 3$, did not participate in the first round. Entries in the $j = 2$, 3, and 4 sections of the table follow in similar fashion.

Consider the second row of Table 8, in which the H2H match-up involves a round $j = 1$ match-up between players A and B, with A being the first player in the match-up, whose score is drawn from record $k = 1$ of Table 2, and B being the second, whose score is drawn from Table 2 record $k = 2$. Then, subtract equation (2.2) from equation (2.1), giving:

$$\begin{aligned} 68 - 71 &= \mu_1 + \delta_1 + \epsilon_{1,1} - \mu_2 - \delta_1 - \epsilon_{2,1} \\ -3 &= \mu_1 - \mu_2 + \epsilon_{1,1} - \epsilon_{2,1} \\ -3 &= \alpha_1 + \beta_2 + \xi_{\{1,2\},1}, \end{aligned} \tag{3}$$

where $\alpha_1 = \mu_1$, $\beta_2 = -\mu_2$, and $\xi_{\{1,2\},1} = \epsilon_{1,1} - \epsilon_{2,1}$.

Following (3), the equations implied by the first four entries in Table 8 are as follows:

$$0 = \alpha_1 + \beta_1 + \xi_{\{1,1\},1} \tag{4.1}$$
$$-3 = \alpha_1 + \beta_2 + \xi_{\{1,2\},1} \tag{4.2}$$
$$1 = \alpha_1 + \beta_4 + \xi_{\{1,4\},1} \tag{4.3}$$
$$1 = \alpha_1 + \beta_5 + \xi_{\{1,5\},1} \tag{4.4}$$

Those implied by the second four entries are:

$$3 = \alpha_2 + \beta_1 + \xi_{\{2,1\},1} \tag{4.5}$$
$$0 = \alpha_2 + \beta_2 + \xi_{\{2,2\},1} \tag{4.6}$$
$$4 = \alpha_2 + \beta_4 + \xi_{\{2,4\},1} \tag{4.7}$$
$$4 = \alpha_2 + \beta_5 + \xi_{\{2,5\},1}, \tag{4.8}$$

and those implied by the last four entries are:

$$-2 = \alpha_5 + \beta_2 + \xi_{\{5,2\},4} \tag{4.47}$$
$$-2 = \alpha_5 + \beta_3 + \xi_{\{5,3\},4} \tag{4.48}$$
$$-4 = \alpha_5 + \beta_4 + \xi_{\{5,4\},4} \tag{4.49}$$
$$0 = \alpha_5 + \beta_5 + \xi_{\{5,5\},4}, \tag{4.50}$$

or more generally, for all k_1 and k_2 within the 50 rows of Table 8,

$$s(k_1) - s(k_2) = \alpha_{i(k_1)} + \beta_{i(k_2)} + \xi_{\{i(k_1),\ i(k_2)\},j} \tag{5}$$

As in the estimation of equation (1), the estimation of equation (5) requires that one of the α_i or β_i values be set to zero. For convenience, β_1 is set to zero. Inasmuch as $\alpha_i = \mu_i$ and $\beta_i = -\mu_i$, setting $\beta_1 = 0$ also sets α_1 to zero. As such, when properly estimated, $\beta_i = -\alpha_i$ for all i, with each α_i representing an estimate of the difference between player i's skill and that of player $i = 1$, and each β_i representing an estimate of the negative of the difference between the estimate of the difference between player i's skill and that of player $i = 1$.

Table 4 summarizes α_i values as estimated via H2H regression equation (5). Note that these estimates are of the same order of magnitude but slightly different from those estimated via standard fixed effects regression as summarized in the "A" column of Table 3. And, similarly, values in the final column of Table 4, which represent the difference in estimated skill of each player relative to that of the best player, player E, are, again, of the same order of magnitude by slightly different than corresponding values estimated via standard fixed effects regression. Although not entirely clear from the description of the TUGR system as presented on its website, I believe the values reported in the final column of Table 4 correspond to what the TUGR system would produce if applied to the data in Table 2. The remainder of this paper follows as if this is in fact the case.

Table 4: Player Skill Differences as Estimated via Head-to-Head Regression Equation (5)

Player		Relative to Player	
		A	E
Name	Index	(α_i)	($\alpha_i - \alpha_5$)
A	1	0.00	1.20
B	2	1.44	2.64
C	3	1.60	2.80
D	4	0.66	1.86
E	5	-1.20	0.00

2.3. The Head-to-Head Model as a Weighted Broadie-Rendleman OLS Regression

Consider H2H equations (4.1)-(4.50), which support equation (5), along with equations (2.1)-(2.14), which support equation (3). Together, it is clear that each of equations (2.1)-(2.4) enter the α portion of equation (5) four times, each of equations (2.5)-(2.7) enter three times, each of equations (2.8)-(2.10) enter three times, and each of equations (2.11)-(2.14) enter four times. Or, more generally, each of equation (2.1)-(2.14) enter the alpha portion of equations (4.1)-(4.50) $n(j)$ times, where $n(j)$ represents the number of players who participated in the round associated with a given equation, (2.1)-(2.14). The same also holds in connection with the β portion of equations (4.1)-(4.50).

By construction, the β portion of equation (5) is identical to the negative of the α portion of the same equation. As such, it is only necessary to estimate the α portion, and this can be accomplished by simply estimating a weighted OLS version of Broadie-Rendleman equation (1), where the weights are the number of players who participated in the round associated with a given scoring observation. Using the scoring data summarized in Table 2, the estimation of the weighted regression produces results identical to those shown in Table 4, with similarly identical results produced when using larger-scale more realistic data sets similar to those used by the OWGR, TUGR, Data Golf, and, perhaps, other organizations that are currently producing golf rankings.

3. Estimation of a TUGR-Type Model in a more Realistic Larger-Scale Data Set

3.1. Player Skill Estimates

The analysis in this section employs the 18-hole scoring data for 2009-2010 used in connection with Broadie and Rendleman (2013), which includes scores in almost all stroke-play events over the period 2009-2010 conducted on the following tours (courtesy PGA TOUR): PGA TOUR, European

Tour, Asian Tour, Australasia Tour, Sunshine Tour, Japan Tour, Nationwide Tour and Challenge Tour.[3] If an event is played on more than one course and a course identifier is available in connection with each 18-hole score, the resulting round-course interaction (or combination) is considered to be a unique round. Unlike Broadie and Rendleman (2013), no limitations are placed on the number of scoring observations per player, since each scoring observation, other than those of players who record only a single score, should be informative.[4] However, once rankings based on weighted OLS regression are determined, one might want to exclude all players from the rankings themselves with fewer than "X" observations.

The data include 160,264 18-hole scoring observations and 18,478,622 H2H match-ups over 4,767 players. Table 5 summarizes players' skill estimates relative to that of the best player, Tiger Woods, for the top-20 players as estimated via the α portion of equation (5), with identical results relative to Woods produced using a weighted version of equation (1). Both regressions were run in R using the feols() function within the "fixest" regression package, with coefficient estimates and standard errors as reported below in Section 3.2 verified by applying R's standard built-in regression function, lm(), to the small-scale data set shown in Table 2.

[3] A tiny fraction of the non-PGA TOUR scoring data is incomplete. Since the exclusions do not appear to be biased, and since a small amount of data is involved, the effect on the results reported here should be negligible.

[4] A scoring observation of a player who records only one score in a given data set is known as a "singleton" and has no effect on the estimation of the various α_i values in equation (5) or, equivalently, the various μ_i and δ_j values in a weighted version of equation (1).

Table 5: Top-20 Players, 2009-2010, as Estimated via TUGR-Type Fixed Effects Regression

Player	N Rounds	Estimated Skill vs. Woods	Rank
Woods, Tiger	115	0.00	1
Stricker, Steve	151	0.35	2
Westwood, Lee	154	0.57	3
Furyk, Jim	157	0.63	4
Casey, Paul	136	0.79	5
Kaymer, Martin	156	0.79	6
Donald, Luke	188	0.90	7
Goosen, Retief	209	0.94	8
Mickelson, Phil	149	1.00	9
McIlroy, Rory	184	1.00	10
Johnson, Dustin	170	1.05	11
Kuchar, Matt	177	1.06	12
Watney, Nick	182	1.06	13
Johnson, Zach	183	1.10	14
Els, Ernie	197	1.12	15
McDowell, Graeme	178	1.14	16
Poulter, Ian	153	1.16	17
Allenby, Robert	184	1.20	18
Mahan, Hunter	180	1.21	19
Villegas, Camilo	171	1.22	20

3.2. Standard Errors

Normally, the feols() regression function does not compute standard errors of the various fixed effect estimates nor does it compute an adjusted R^2 value. However, at the expense of significantly more execution time, standard errors and adjusted R^2 values can be computed by formulating feols() in a mathematically equivalent but slower form.

When computing standard errors, equation (1) is run two different ways, first in unweighted form, equivalent to the Broadie-Rendleman model, and then in weighted form, equivalent to what I believe to be mathematically equivalent to the TUGR model. Typically, when run in weighted form, the adjusted R^2 is slightly higher, in this case 0.340 vs. 0.333. However, as shown in Table 6, which lists the top 20 players as ordered by the number of rounds played, the players for whom precise estimates of skill and resulting rankings tend to matter the most, when estimated in connection with an unweighted version of equation (1), player skill estimates relative to that of Tiger Woods tend to be slightly higher relative to those estimated via a weighted version of equation (1) and standard errors tend to be slightly lower. Thus, there is no evidence in this data, nor in any other data sets I have employed, that my version of the TUGR model produces better player skill estimates than a standard unweighted Broadie-Rendleman fixed effects regression model.

Table 6: Skill Estimates vs. Woods and Standard Errors of Skill Estimates as Estimated in Connection with Unweighted and Weighted Versions of Equation (1)

		Unweighted		Weighted		Unweighted vs. Weighted	
Player	N Rounds	Skill vs. Woods	Standard Error	Skill vs. Woods	Standard Error	Skill vs. Woods	Standard Error
Senden, John	221	1.698	0.333	1.612	0.363	0.087	-0.030
Ishikawa, Ryo	214	2.141	0.341	2.043	0.375	0.099	-0.034
de Jonge, Brendon	214	2.350	0.336	2.246	0.364	0.104	-0.028
Hiratsuka, Tetsuji	214	2.742	0.343	2.581	0.373	0.161	-0.031
McGrane, Damian	211	2.402	0.340	2.330	0.364	0.072	-0.024
Drysdale, David	210	2.687	0.340	2.590	0.363	0.097	-0.023
Goosen, Retief	209	1.092	0.337	0.942	0.373	0.150	-0.036
Green, Nathan	208	2.926	0.337	2.636	0.365	0.289	-0.028
Garrido, Ignacio	207	2.437	0.341	2.326	0.364	0.111	-0.023
Van Pelt, Bo	204	1.540	0.338	1.403	0.371	0.138	-0.032
Davis, Brian	204	2.536	0.339	2.459	0.367	0.076	-0.029
Appleby, Stuart	203	2.520	0.338	2.331	0.365	0.189	-0.027
Howell III, Charles	202	1.867	0.339	1.784	0.369	0.083	-0.029
Jaidee, Thongchai	202	1.945	0.341	1.818	0.368	0.127	-0.027
Petrovic, Tim	202	2.329	0.339	2.238	0.368	0.091	-0.028
Jacquelin, Raphael	202	2.422	0.342	2.320	0.365	0.102	-0.023
Jimenez, Miguel A.	200	1.769	0.342	1.616	0.371	0.153	-0.029
Merrick, John	200	2.541	0.340	2.370	0.369	0.171	-0.029
Levin, Spencer	199	2.354	0.341	2.197	0.367	0.157	-0.026
Fujita, Hiroyuki	199	2.212	0.348	2.217	0.382	-0.004	-0.034

It should be noted that a major component of the new OWGR system is Strokes Gained World Ratings (SGWRs), which, if not for the use of the OWGR's standard time-weighting scheme, would produce relative skill estimates and rankings identical to those associated with an unweighted version of equation (1) as applied to the same underlying scoring data. As illustrated above in Table 6, with lower standard errors, non-time-weighted SGWR should produce more accurate relative skill estimates among the world's most active players than what I believe to be the TUGR system. Whether SGWR in time-weighted or non-time-weighted form represents the best way to estimate player skill remains an open question, but based on the results summarized in teh table, it would appear that non-time-weighted SGWR should produce more accurate player skill estimates and rankings than my version of TUGR.

3.3. Execution Time

Although the execution time associated with actual TURG calculations is unclear, I believe TUGR's use of a "powerful cloud-based optimizer" and references within the TUGR website to potentially high execution costs suggests that the execution time reguired to make actual TUGR calculations is non-negligible. By contrast, as shown in Panel B of Table 7 below, the time required to execute a weighted version of equation (1) (my version of the TUGR model) in R is almost negligible, 1.3 seconds. (Execution times in connection with an unweighted version of equation (1) are essentially the same.) On the other hand, it takes a total of 127.9 seconds to execute my version of the TUGR model by first assembling H2H match-ups and then estimating equation (5) directly (Panel A). As shown in Panel C, execution time of 149.3 seconds is required if one wants to obtain standard errors as computed in connection with unweighted and weighted versions of the Broadie-Rendleman model.

Table 7: Execution Times (Seconds) in R

Panel A: H2H Model

Time to construct 18,478,622 H2H match-ups		78.2
Time to run H2H regression equation (5), determine the best player, and assemble results relative to the best player.		49.6
	Total	127.8

Panel B: Weighted Regression Model

Time to compute regression weights		0.2
Time to determine best player via weighted version of regression equation (1)		0.7
Time to re-run weighted version of regression equation (1) with the best player omitted		0.4
	Total	1.3

Panel C: Standard Errors

Time to compute standard errors for weighted and unweighted versions of regression equation (1)	149.3

REFERENCES

Broadie, M. and R. J. Rendleman, Jr. (2013): "Are the Official World Golf Rankings biased?" *Journal of Quantitative Analysis in Sports*, 9, 127–140.

Table 8: All Possible Head-to-Head (H2H) 18-Hole Scoring Match-ups

Row	Round Index j	$n(j)$	k_1	k_2	First Player Name(k_1)	Second Player Name(k_2)	First Player Index $i(k_1)$	Second Player Index $i(k_2)$	First Player Score $s(k_1)$	Second Player Score $s(k_2)$
1	1	4	1	1	A	A	1	1	68	68
2	1	4	1	2	A	B	1	2	68	71
3	1	4	1	3	A	D	1	4	68	67
4	1	4	1	4	A	E	1	5	68	67
5	1	4	2	1	B	A	2	1	71	68
6	1	4	2	2	B	B	2	2	71	71
7	1	4	2	3	B	D	2	4	71	67
8	1	4	2	4	B	E	2	5	71	67
9	1	4	3	1	D	A	4	1	67	68
10	1	4	3	2	D	B	4	2	67	71
11	1	4	3	3	D	D	4	4	67	67
12	1	4	3	4	D	E	4	5	67	67
13	1	4	4	1	E	A	5	1	67	68
14	1	4	4	2	E	B	5	2	67	71
15	1	4	4	3	E	D	5	4	67	67
16	1	4	4	4	E	E	5	5	67	67
17	2	3	5	5	A	A	1	1	71	71
18	2	3	5	6	A	B	1	2	71	72
19	2	3	5	7	A	D	1	4	71	72
20	2	3	6	5	B	A	2	1	72	71
21	2	3	6	6	B	B	2	2	72	72
22	2	3	6	7	B	D	2	4	72	72
23	2	3	7	5	D	A	4	1	72	71
24	2	3	7	6	D	B	4	2	72	72
25	2	3	7	7	D	D	4	4	72	72
26	3	3	8	8	B	B	2	2	73	73
27	3	3	8	9	B	C	2	3	73	75
28	3	3	8	10	B	D	2	4	73	72
29	3	3	9	8	C	B	3	2	75	73
30	3	3	9	9	C	C	3	3	75	75
31	3	3	9	10	C	D	3	4	75	72
32	3	3	10	8	D	B	4	2	72	73
33	3	3	10	9	D	C	4	3	72	75
34	3	3	10	10	D	D	4	4	72	72
35	4	4	11	11	B	B	2	2	71	71
36	4	4	11	12	B	C	2	3	71	71
37	4	4	11	13	B	D	2	4	71	73
38	4	4	11	14	B	E	2	5	71	69
39	4	4	12	11	C	B	3	2	71	71
40	4	4	12	12	C	C	3	3	71	71
41	4	4	12	13	C	D	3	4	71	73
42	4	4	12	14	C	E	3	5	71	69
43	4	4	13	11	D	B	4	2	73	71
44	4	4	13	12	D	C	4	3	73	71
45	4	4	13	13	D	D	4	4	73	73
46	4	4	13	14	D	E	4	5	73	69
47	4	4	14	11	E	B	5	2	69	71
48	4	4	14	12	E	C	5	3	69	71
49	4	4	14	13	E	D	5	4	69	73
50	4	4	14	14	E	E	5	5	69	69